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# Development of Transonic Area-Rule Methodology

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**This study presents an innovative method for area ruling in the transonic regime. The method applies a weighting function to the sonic area rule that generally accounts for the physical nature of transonic flow. In sonic flow, changes in pressure are communicated with negligible dissipation along Mach planes. As a result, drag becomes a strong function of the cross-sectional area development of the aircraft. Transonic flow has the added complexity of mixed subsonic and supersonic regions. In this flow, the communication between the aircraft fuselage and its external parts has dissipation due to embedded subsonic regions. Therefore, the sonic area rule no longer strictly applies. The new transonic area-rule methodology, described in this article, utilizes a weighting function that adjusts for the effects of the mixed flows. The shaping methods resulting from this new transonic area-ruling technique are much less severe than the standard sonic area-ruling method and require substantially less body modification. Furthermore, the new transonic area-ruling technique maintains drag rise delays that are the same as the traditional sonic area rule.**

## Nomenclature

$A$ = stream-tube area
$C_D$ = drag coefficient
$M$ = local Mach number
$t_{parm}$ = wingtip parameter
$v$ = velocity
$\Delta C_D$ = wave drag coefficient

## Introduction

THE limiting subsonic speed at which high-performance transport and business jet aircraft fly is often set by drag rise due to compressibility effects. Delaying this transonic drag rise will potentially allow the design of more efficient and faster subsonic aircraft. One example of efforts in industry to reduce or avoid drag rise is the Learjet 60 (Fig. 1).[1] Note that the fuselage is modified significantly at the wing juncture. A new method of shaping the fuselage in order to delay drag rise is presented in this article. The new method modifies the traditional sonic area rule, a well-recognized technique for delaying drag rise.

Some of the first glimpses into the physics of the area rule were provided by Hayes.[2] The linearized equations he developed for predicting supersonic wave drag showed that as the Mach number approached unity the wave drag calculation simplified to that of a body of revolution. Whitcomb presented the area rule and showed, experimentally, that drag rise could be delayed and reduced in magnitude through sonic area ruling of the fuselage.[3] The theoretical supersonic area rule was presented the following year, 1953.[4] The supersonic area rule has been validated many times and other methods for the supersonic regime have been explored, including the moment-of-area-rule[5] and the pressure-field-rule.[6] Area ruling in the transonic regime has had little refinement since its initial development, although some work was done using the pressure rule.[7,8] Traditionally, transonic design involving the area rule has utilized the standard sonic area-ruling method and iterative numerical techniques that require extensive engineering judgment.

## Traditional Area-Ruling Concept

The sonic area rule is a far-field method of predicting and understanding wave drag due to shock losses.[3] It is based on the idea of perfect pressure disturbance communication between the wing, or other external features, and the fuselage. It proposes that a body of revolution with the same axial development of cross-sectional area will have a wave drag that is similar to the original configuration.[3] This idea was validated through wind-tunnel tests in the 1950s.[3]

The area rule can be viewed theoretically from the compressible inviscid equations. The simplest way of doing this is through the one-dimensional equations. Equation (1) is a combination of Euler's one-dimensional equation, conservation of mass, and the definition of the speed-of-sound. As $M$ approaches unity, the stream-tube areas essentially become

$$\frac{dv}{v}(1 - M^2) + \frac{dA}{A} = 0 \tag{1}$$

invariant.[3] The idea of constant stream-tube area is a theoretical insight into how the area rule works. Area changes in the configuration along a Mach 1 plane create changes in the flow that are felt by all configuration surfaces that intersect that plane.

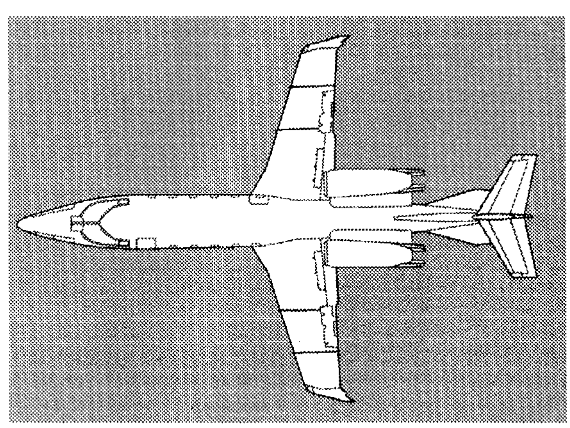

Fig. 1 Top view of the Learjet 60.

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The key point and justification to the sonic area rule is that pressure changes created by a change in cross-sectional area at any location on the aircraft are experienced by all the surfaces of the aircraft that intersect that same plane. Dissipation of the disturbances are considered to be negligible.

When an aircraft is area-ruled, the development of the cross-sectional area intersected by the Mach planes is optimized. The area development is generated by integrating the area of the aircraft that intersects the Mach plane at a series of stations along the fuselage axis. In this study, all of the area ruling is done near Mach 1 or less, so the Mach planes are always considered to be perpendicular to the fuselage axis. The area rule is applied by removing area or volume from the fuselage in order to account for the area or volume of the wings. In general, area ruling can also be done for any other external part of the aircraft.

## Transonic Area-Ruling Concept

During a forum on transonic design at NASA Langley Research Center, Whitcomb commented that he had applied a linear weighting function to the sonic area rule as a rule-of-thumb for transonic design, but had never investigated it formally. The method described in this study is the result of the first formal investigation and extension of his ideas.

The method presented in this article is based on the degree of communication between the flows on the wing and the fuselage. This communication changes in the transonic regime because of the mixed supersonic and subsonic flow. In transonic flow, dissipation of disturbances occurs in the subsonic regions and the stream-tube areas are no longer invariant. As a result of this dissipation, it is erroneous to subtract the total wing volume from the fuselage. Part of the pressure changes created by the indentations in the fuselage is dissipated before it reaches the wings by passing through embedded subsonic regions. To account for the majority of this dissipation, the new transonic method applies a weighting function to the sonic area rule that modifies volume removed from the fuselage. Only the volume that will relieve the flow on the wing should be removed. This minimizes the volume removed from the fuselage and still maintains the drag rise delay.

The weighting functions used in this study are applied during the integration of the area of the wing that is intersected by a given Mach 1 plane. Area at the wingtip is given less value than area near the wing root (Fig. 2). The weighted wing area is then subtracted from the fuselage area that is intersected by this same Mach plane. This procedure is repeated at a series of locations along the fuselage axis, resulting in a net volume removal from the fuselage.

## Investigative Approach

The investigation of this new method could have been carried out in two different ways: 1) experimentally or 2) computationally [computational fluid dynamics (CFD)]. A CFD approach was chosen for this initial study because it allowed a fairly quick analysis of the new concept. As described next, the chosen CFD method was validated with existing experimental results to ensure that it could properly predict trends in wave drag through the transonic regime when area ruling was applied. Because the code was successful in these predictions, it was used to investigate the new area-ruling technique.

### CFD Method

An unstructured grid Euler code, USM3D, was used for the study.[9] The Euler code contained the necessary physics to show the correct trends in wave drag. The adaptability of the unstructured grid was a big advantage since the study included a large number of fuselage shapes. One disadvantage was the lack of viscous effects, which resulted in downstream shifts of the transonic shocks that occurred on the wings.[10–12] These shifts have some effect on the drag integration. Therefore, before USM3D could be used to explore the effects of the weighting functions it was compared with experimental area-rule data.

### Convergence Criteria

The convergence criteria for the CFD code included: iterating until the residual error had reduced 3.5 orders in magnitude; ensuring that the slope of the residual vs number of iterations continued to decrease; visually inspecting $C_D$ vs number of iterations; and comparing grids of different densities in order to ascertain convergence.

To ascertain if the grids were sufficiently dense, they were checked by comparing grids of different densities. The average error between solutions due to grid density on the delta-wing models was 0.6–1.2 drag counts and on the swept-wing model it was 0.07–0.3 drag counts.

### Choice of Experimental Data

For the evaluation of the CFD code to be relevant, the experimental data characteristics were to meet the following criteria: drag results from the beginning of the transonic regime to Mach 1.0 were essential, simple model configurations were desired so that the salient physics could be seen, a configuration that was similar to a high-performance subsonic transport or business jet was desired, and data that provided comparisons between normal and area-ruled configurations were needed.

Data of this type were scarce and some of the best available were from Whitcomb's sonic area-rule experimental results.[3] These data met all of the previous criteria. The configurations that were used from his experiment included a delta-wing model and a swept-wing model.[3] The delta-wing model provided a large change in wave drag between the normal model and the area-ruled model (Fig. 3). A large change in wave drag permitted trends in the predictions of the code to be seen. The swept-wing model was more representative of a business jet configuration although it is much more slender (Fig. 4).

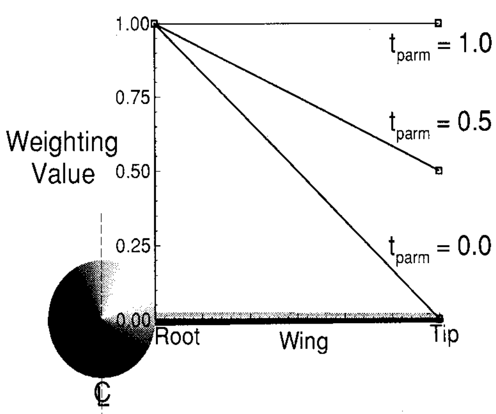


**Fig. 2 Weighting function concept. Front view of model.**

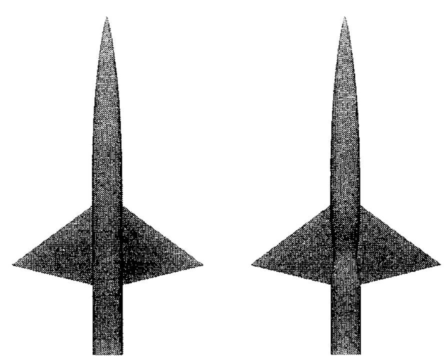

**Fig. 3 Delta-wing experimental model.**

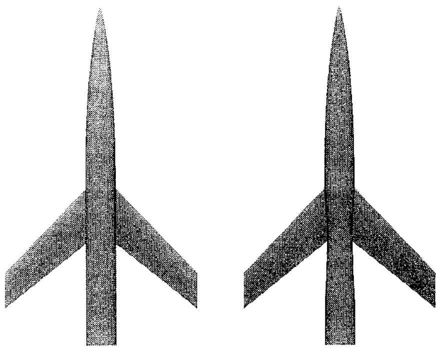

**Fig. 4 Swept-wing experimental model.**

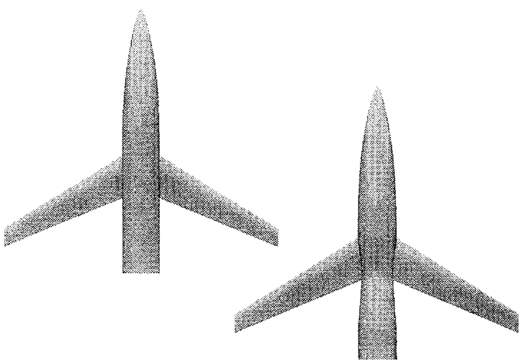

**Fig. 5 Business jet model, no area rule and sonic area rule.**

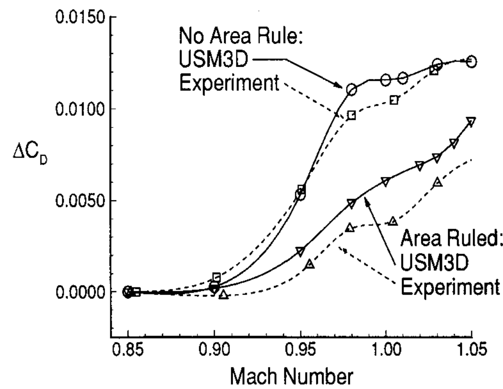


**Fig. 6 $\Delta C_D$ plot on the no area rule and area-ruled delta-wing model.**

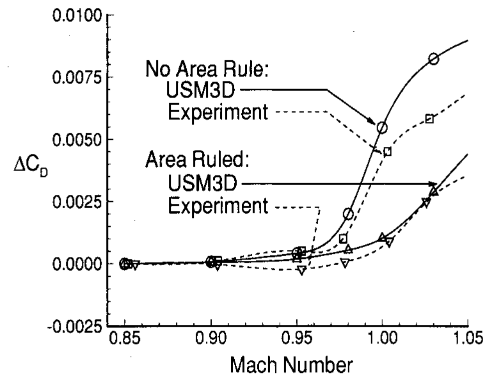


**Fig. 7 $\Delta C_D$ results on the no area rule swept-wing model.**

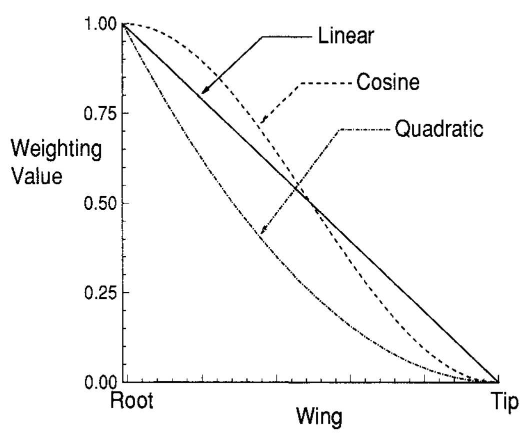


**Fig. 8 Various weighting functions.**

### Business Jet Model

In order to investigate a configuration that more closely represented a business jet aircraft, a numerical model was created. This model was a derivative of the swept-wing experimental configuration (Fig. 5). The modifications included: the wing sweep was changed from 45 to 30 deg, the wing thickness ratio was modified from 6 to 12%, the wingspan was increased by 83.3%, the wing-taper ratio was decreased from 0.64 to 0.44, and the body length-to-diameter ratio was changed from 11.5 to 7.2.

## Comparison with Experiment

Experimental and numerical graphs of $\Delta C_D$ vs Mach number were compared. $\Delta C_D$ is the approximate wave drag, it represents a change in drag from a reference drag value. This reference drag is found by inspecting the drag curve and is located just before drag rise begins. Since the majority of drag increase in the transonic regime is due to wave drag, $\Delta C_D$ is a good approximation of wave drag. It is the variable used for comparison in Ref. 9. $\Delta C_D$ is also used for the comparison in this study because area ruling only reduces wave drag. $\Delta C_D$ is the quantity used throughout the study.

### Delta-Wing Model

The first comparison that was made between the USM3D and experimental results was on the normal delta-wing model (Fig. 3). Figure 6 shows that the correlation of USM3D with the experimental $\Delta C_D$ was exceptionally good. By inspection, it can be seen that maximum error occurred near Mach number 1, where experimental results were questionable.

The area-ruled delta-wing model correlated fairly well with the experimental values in the Mach number range of 0.85–1.00, but did not correlate nearly as well throughout the full Mach number range (Fig. 6). The increased error was attributed to the lack of a boundary layer, or viscous effects, in the theoretical model. Also, separation due to the severe shaping of the fuselage would cause the boundary layer to fill the indented portion of the fuselage, which would affect the area-ruling seen by the flow. However, the theoretical results followed the experimental trends.

### Swept-Wing Model

The swept-wing models correlated well with the experiment in the range of interest, Mach 0.85–1.00 (Fig. 7). It was interesting to note that, due to the area development of this configuration, the drag rise was delayed until near Mach 0.97.

The good correlation with experimental data and the correct predictions in the wave-drag trends provided the confidence to use USM3D as an analysis tool for the evaluation of new transonic area-ruling methods.

## Transonic Weighting Functions

The weighting functions used in this study are simple algebraic equations that are a function of wingspan and $t_{\mathrm{parm}}$. Different functions of wingspan have been included in the investigation to determine the degree of communication of each part of the wing with the fuselage (Fig. 8). In a general case, the weighting function will likely depend on many more variables. Although not investigated in this study, it appears that the weighting function should also be affected by the wing sweep, Mach number, angle of attack, and body length-to-diameter ratio.

## Linear Weighting Function

The linear weighting function consisted of a simple linear distribution of values from the wing root to the wingtip. The wing root was set to a value of one and the $t_{parm}$ was set to a value between zero and one. If $t_{parm}$ were equal to zero, then a delta area midway between the root and the tip would only be integrated at half its value (Fig. 9). The $t_{parm}$ value was varied to see the degree of influence that indentations in the fuselage had on the outboard sections of the wing.

### Application to Delta-Wing Model

The results indicated that a delta-wing aircraft can be area ruled in a much less severe fashion than traditionally expected and still maintain nearly the same drag-rise reduction. The best example of this was when the $t_{parm}$ was equal to 0.0 (Fig. 10). In this case, the volume removal from the area-ruled section was about 75% of what sonic area-rule prescribes, but the drag rise reduction was nearly the same. Figure 11 shows how well the $t_{parm}$ equal to the 0.0 case correlated with the sonic area ruled $\Delta C_D$ data.

### Application to Swept-Wing Model

Results similar to that of the delta-wing model were obtained with the swept-wing model, except that a $t_{parm}$ of 0.25 was required to yield a drag reduction similar to the sonic area ruled results (Figs. 12 and 13). The volume removal from the fuselage was about 68% of the sonic case.

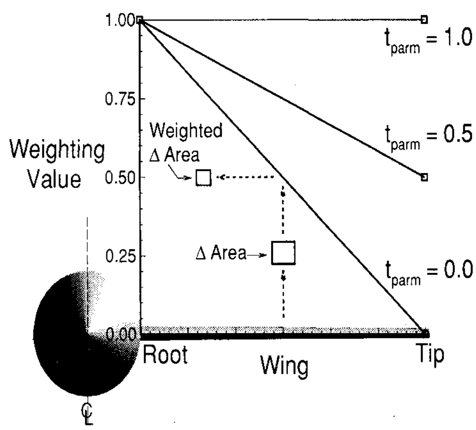


**Fig. 9 Linear weighting function concept. Front view of model.**

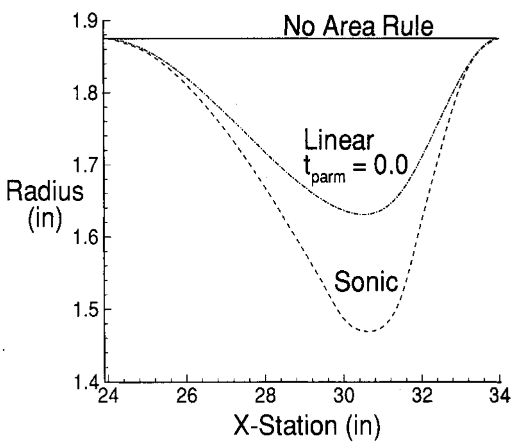


**Fig. 10 Delta-wing model: weighted area rule vs sonic area rule.**

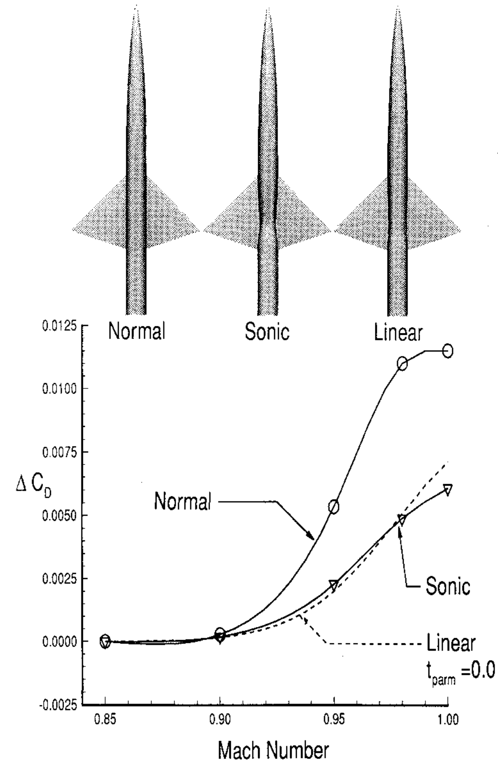


**Fig. 11 Delta-wing model $\Delta C_D$ results with linear weighting function applied.**

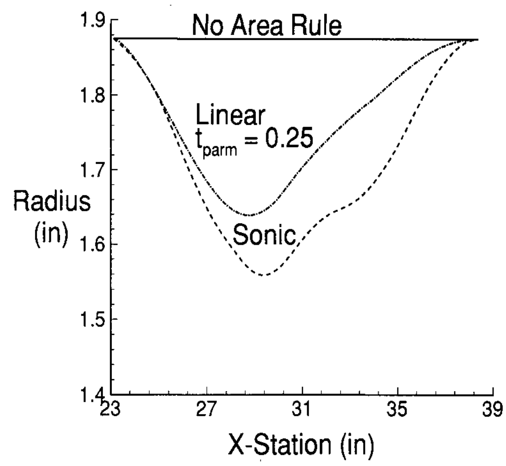


**Fig. 12 Swept-wing model: weighted area rule vs sonic area rule.**

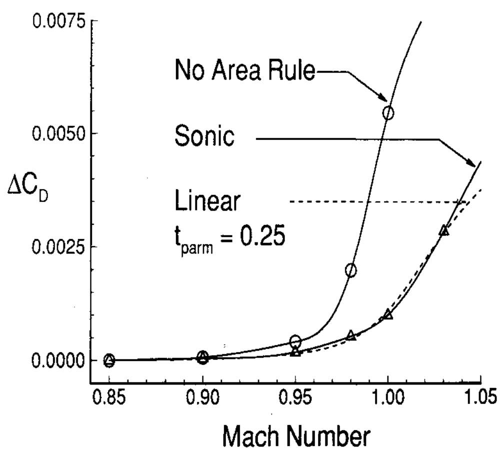


**Fig. 13 Swept-wing model $\Delta C_D$ results with linear weighting function applied.**

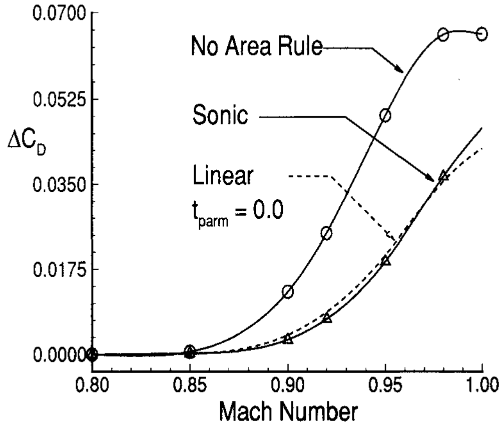


**Fig. 14 Business jet model $\Delta C_D$ results with linear weighting function applied.**

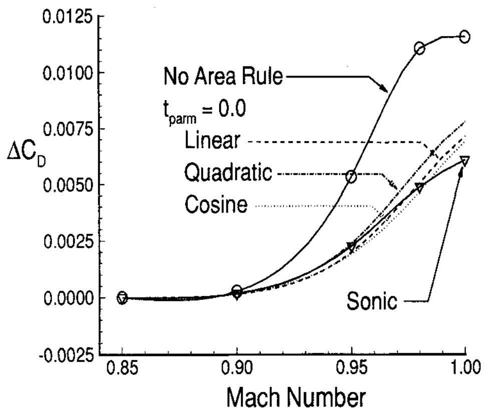


**Fig. 15 Delta-wing model: weighted area rule $\Delta C_D$ comparison.**

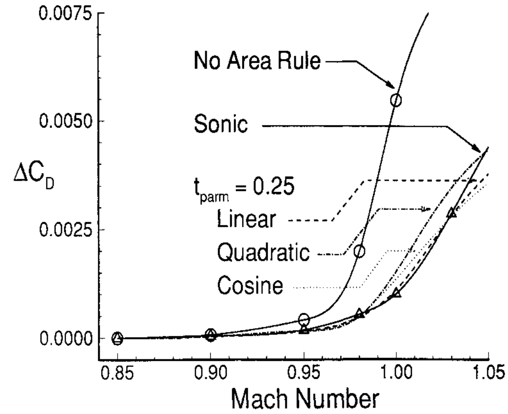


**Fig. 16 Swept-wing model: weighted area rule $\Delta C_D$ comparison.**

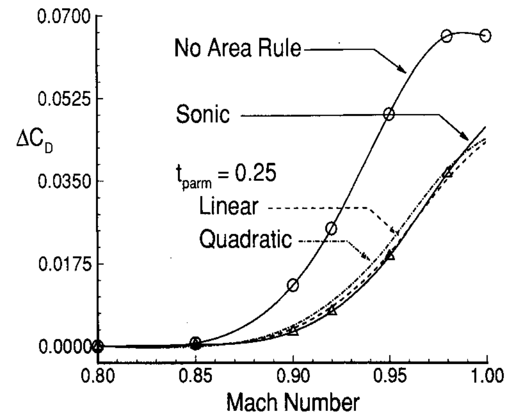


**Fig. 17 Business jet model: weighted area rule $\Delta C_D$ comparison.**

#### Application to Business Jet Model

The business jet model results reflected the benefits that modern business jets could enjoy by employing this weighting method (Fig. 14). For $t_{parm}$ equal to zero, the volume removal from the fuselage was about 64% of what sonic area rule prescribes. Note the drastic increase in the wave drag levels with this much less slender configuration as compared to the experimental model (Figs. 4, 5, 13, and 14).

## Other Weighting Functions

The results from the quadratic and the cosine weighting functions were computed for the swept-wing and delta-wing models. Only the results using the quadratic weighting function have been computed for the business jet model. The quadratic function presented the greatest advantage because its application resulted in the smallest amount of volume removal from the fuselage, and yet, it still maintained the majority of the drag rise delay (Figs. 15–17). The volumes removed from the delta-wing, swept-wing, and business jet fuselages were 60, 55, and 61%, respectively, of what sonic area rule dictated.

The cosine function represented greater interaction between the wing and the fuselage near the root of the wing and significantly less interaction near the tip of the wing (Figs. 15 and 16). It maintained the drag rise delay, but did not reduce the amount of volume removed from the fuselage as significantly as the quadratic function.

## Future Considerations

This study of the modified transonic area rule only considers first-order methods and applies them in a global sense. As this technique becomes more refined, it may also prove useful in a more localized sense. For example, the juncture of nacelles with wing or the fuselage could possibly be locally area ruled using the weighting method. General transonic area-rule design should consider both local and global effects when doing body modifications.

This study is done at zero angle of attack, but future work should consider the effects of lift. At lifting conditions the stream tubes expand and the equivalent wing volume increases; this will affect the weighting function.[13,14]

Boundary-layer and shock interaction are completely neglected in the Euler solutions, but the boundary layer has a significant effect on the shock location.[10–12] Also, the onset of drag rise on relatively thick wings is due to boundary-layer separation at cruise conditions.[15] The boundary layer should be considered as this transonic methodology is developed further.

Most importantly, future work should include wind-tunnel and flight experiments. The results that have been obtained in this study are very promising, but also very preliminary. Wind-tunnel tests of parametric model variations in area ruling should aid in further development of this method.

## Conclusions

This study has emphasized some important points about area ruling in the transonic regime. These points include the following:

1) The transonic area-ruling methodology discussed in this article provides the benefits of drag rise delay with significantly less body modification than traditionally expected. Drag rise delays that match the traditional sonic area rule can be obtained by modifying the aircraft to only 60% of what sonic area rule prescribes.

2) The dissipating effect of the subsonic regions in the transonic regime significantly influences the approach to area ruling. This method adjusts for these subsonic regions by

modifying the traditional sonic area-rule methodology with a geometric weighting function.

3) The methodology developed in this study, while a first-order method, shows promise as an engineering design tool. Recommendations for extending the method are included in this article.

## Acknowledgments

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